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Minimization of transport costs in an industrial company through linear programming

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Abstract. Businesses and industries are very interested in optimizing processes and minimizing costs. One of the transportation models or problems to optimize relates to the case where the product output can be transported from the manufacturing plant to some warehouses or customers. The transport problem is a case of the linear programming problem. Linear programming algorithms have been used to solve the most difficult optimization problems. Linear programming has been used to manage the problems related to personnel assignment, engineering, distribution, banking, education, oil, transportation, etc. It has been widely used in various fields, like transport, telecommunications, health services, construction, public services, industry, etc.
The main purpose of the paper is to look at the problem of linear programming in detail by considering an example and try to solve the problem. The purpose of the transport problem in our case is to minimize the overall cost of transport from origin to destination by meeting supply and demand limits, in order to increase sales profit. This paper aims to determine the problem of minimizing the total cost of transport by determining the optimal distribution of the product, from the 2 sites of production to the 9 major distributors that are geographically dispersed. Management Science and Operational Research will be used to find the best solution for the transportation problem.

1. Introduction

Transportation problems are an integral part of linear programming, which accompanies us with daily activities in our life and mostly deals with the logistics sector. It helps to solve transportation problems from one origin to a relevant destination. The basic objective of the problem is to minimize transport costs from origin to destination.

In order to make strategic decisions the model is used for selecting the optimal transport routes in order to divide the production of different origins into many warehouses or centers of distribution. The transport model can be used in the other hand in making localization decisions. When two or more locations are being considered, the model helps in order to find a manufacturing plant, a new facility, or an office. Total costs of transport, delivery and production should be minimized by applying the model. Transport models play an important role in logistics and supply chain. The objective function is the minimized or maximized function. We have a linear programming problem in cases when the linear system model and objective functions are both linear equations. Moreover, linear programming algorithms are used to solve the most difficult optimization problems. In a transportation problem, are defined origins, the factories where the products are manufactured and which supply a required quantity of products to a certain number of destinations. The objective in a transportation problem is to fully meet the demand of the goods to the destination within the constraints of the production capacity at the least possible cost. Transportation problems deal with the problem of production planning and transportation given the quantity offered and the demand that different distribution locations and their product customers require.

2. Literature review

Nowadays mathematical methods solve many problems of operational planning for the transportation. Most of these transportation problems us linear programming method. Optimization means using existing technology and resources and at the best way possible. Better planning and execution results in optimization for a lot of problems. Quantitative models and mathematical tools such as linear programming allows a better result. Modern computing equipment can be used in this case.

Ahmed et al. (2017) proposed a new method to obtain an initial basic feasible solution for the transportation problems. The proposed method was illustrated with numerical examples and also a comparative study was done to justify the performance of the proposed method. It was noticed that the performance of proposed method was suitable for solving transportation problems.

Abdul et al. (2012) has shown a new method named ASM Method for finding an optimal solution for many transportation problems. By this method was established a numerical illustration and was yielded the best the result. This method required very simple arithmetical and logical calculation. This method was explored for those decision makers dealing with logistics and supply chain related issues.

Islam et al. (2012) has proposed an algorithm called Incessant Allocation Method to obtain a starting basic feasible solution for the transportation problems. Were solved a several numbers of numerical problems in order to justify the method and the results have shown that the proposed algorithm was effective in solving the problems of transportation.

Nigus and Tripti (2013) have analysed Vogel's Approximation Method and its modification due to Shimshak and Goyal in finding an initial solution to an unbalanced transportation problem. They have suggested a heuristic approach for balancing the unbalanced transportation problem and improving the Vogels Approximation Method.

Bazaraa et al. (2009) has presented concepts and techniques that are illustrated by numerical examples along with insights completed with detailed mathematical analysis and justification.

Muztoba (2014) has examined a real-world application of a transportation problem that involves transporting mosquito coil from company's warehouse to distributor's warehouse is modelled using linear programming in order to find the optimal transportation cost.

3. The variables, objective function, model and the problem with mathematical formulation

Assume a company has m origins and n destinations. A product will be moved from origin m to destination n. Every warehouse or manufacturing plant has a certain level of supply, and each retail market has a certain demand. The cost of transport between each pair of origin and destination is also given and these costs are assumed to be linear.

Total product supply is denoted i = a, where we have i = 1,2,3,4, … . m
The total market demand is denoted j = b, where we have j = 1,2,3,4, … . n
The cost of transport per unit of each product from origin m to destination n is expressed by cij, where we have i = 1,2,3,4, … m and j = 1,2,3,4, … n.

3.1. The variables

The variables in this case of linear programming model of the transport problem will have the values for the number of units sent from an origin m to a destination n.

All the variables are presented by:

Xij- the size of the load from origin m to the destination n.
Where we have i = 1,2,3,4, … m and j = 1,2,3,4, … n. This is a series with m, n variables.

3.2. Objective function
The total cost of transportation is given by cij * xij. This is done when we say that the total functional cost is linear.

Collecting all i's and j's now gives the total shipping cost for all origin-destination pair combinations. The transport problem can be formulated as a linear programming problem model:
Minimization of: $\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}C_{ij}$
Where:
$\boldsymbol{x_{ij}}$ - is the quantity of units that are moved from origin m to destination n.
$\boldsymbol{c_{ij}}$ - is the cost of moving a unit from origin m to destination n.

3.3. Limitations
Limitations are the conditions that compel the necessary capacity and demand to be met. In a problem of transport there is a drawback for each node.
We have:
a1 - Origin capacity indicator m
b1 - destination capacity indicator n

1. The capacity of each supplier can be:
$$\sum_{i=1}^{m} X_{ij} = a\,,\quad where\ i = 1,2,3 \ldots m \tag{1}$$
2. The request for any destination can be:
$$\sum_{j=1}^{n} X_{ij} = b\,,\quad where\ j = 1,2,3 \ldots n \tag{2}$$
3. Condition of non-negativity:
$$X_{ij} \geq 0\ for\ each\ i\ dhe \tag{3}$$

3.4. Transport model Formatting the title
Minimization of:
$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}C_{ij} \ldots \ldots \tag{4}$$
The supply capacity constraint
$$\sum_{i=1}^{m} X_{ij} \leq ai \qquad i = 1,2,3,4, \ldots . m \tag{5}$$
The demand capacity constraint
$$\sum_{j=1}^{n} X_{ij} \geq bj \qquad j = 1,2,3,4 \ldots . n \tag{6}$$
$$X_{ij} \geq 0\ for\ i = 1,2,3,4, \ldots . m,\ j = 1,2,3,4, \ldots . n \tag{7}$$
This is a linear programing with the variables m, n, and with m + n limiting variables and with m, n non-negative variables.
Where:
m = nr. of origins, n = nr. of destinations, ai = capacity of origins (in ton, liter, etc.), bj = capacity of destinations (in tonnes, liters, etc.), Cij = the cost of materials.

The existence of the equation below is a necessary and sufficient condition for the existence of a possible solution to the transport problem:
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{8}$$
When this equation exists, we are dealing with a balanced transportation problem.

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3.5. The problem of unbalanced transportation

If:

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j \quad (9)$$

Transport problems are known as unbalanced transport problems.

Case 1:

The available capacity of the originals is greater than the capacity at the destination required.

$$\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j \quad (10)$$

Case 2:

The available capacity of the origins is less than the capacity at the destination required.

$$\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j \quad (11)$$

4. Representing the transport problem network

There is also a graphical representation of this problem.

Graphically, this problem is often seen as a network of m sources and n destinations, and a set of m, n "directed arcs". This network of transport problems is shown in the following figure.

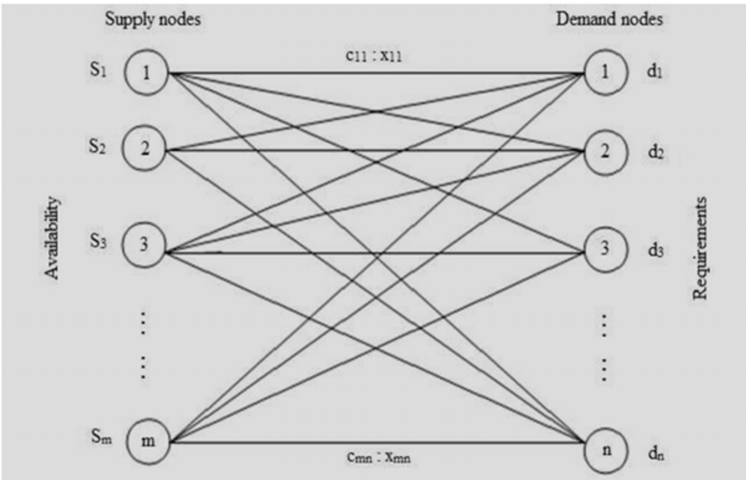

Figure 1. Source and destination nodes in the transport problem network

S1… …Sn → origins

D1… …Dn → destinations

C1… …Cn → cost of transport

The arrows indicate the transport flows from source to destination.

4.1. Degradation of the transport problem

A transport problem is called degraded in the cases when the nr of filled cells is less than the nr of rows plus the nr of columns minus one (m + n - 1). We can be observed degradation during the initial allocation where the first entry in a row or column meets the row and column requirements, or when the added and subtracted values are equal, in cases when is applied the method of Stepping Stone. The basic solution is called degraded, when the transport problem with m origins and n destinations does not have m + n-1 positive base variables. Whenever the number of base cells is not more or equal than m + n - 1, the transport problem is degraded. To solve the degradation problem, are increased the positive variables with a lot of zero variables in such a way as to satisfy the basic variable m + n -1.

4.2. Solution algorithm for transport problem

The solution algorithm has the following steps:

Step 1. Formulation of the problem and place the data in matrix form. The objective function is the total cost of transport. The constraints are the demand and supply to each source and each destination.

Step 2. The basic solution can be reached using one of the methods:

North West Corner Rule, Least Cost Method, Vogel Approximation Method

The solution must meet all supply and demand constraints. The nr of positive partitions must be equal to m + n - 1, where m is the nr of rows and n is the nr of columns.

Step 3. Test the initial solution for optimality with the Stepping Stone Method or Modified Distribution Method

Step 4. Update the solution. Then repeat Step 3 until the optimal solution is reached.

4.3. Transportation Problem Methods

a. North West Corner

The North West Corner Rule is a method to compute the initial feasible solution of the transport problem. The name is given to this method because the basic variables are selected from the extreme left corner.

b. Least Cost Method

In the transport problem the Least Cost Method is used to obtain the initial feasible solution. The allocation begins with the cell which has the minimum cost. The lower cost cells are chosen over the higher-cost cell in order to have the least cost of transport.

c. Vogel's Approximation Method

VAM (Vogel's Approximation Method) is an iterative procedure calculated to find out the initial feasible solution of the transport problem. Here is taken into consideration the transport cost, but in a relative sense.

5. Case study of a transport problem in a company in Albania

This study aims to minimize the total cost of transportation of a company, from the two production sites, S1 and S2 for its many distributors throughout Albania, which are nine. The total factory capacity for S1 and S2 per day is 1736 and 2419 units. The company faces challenges in how to optimally distribute its products to 9 major distributors at a minimal trasportation cost. Each production site has its own supply limit and each customer has a specific demand at a time. The data is based on two manufacturing plants located in two different sites and distributes this manufactured product in 9 geographical regions. We have factories S1-S2 and destinations D1-D9

Table 1. Capacity and demand of sources and destinations

From/ To	D 1	D2	D3	D4	D5	D6	D7	D8	D9	Capacity
S1	90.79	88.21	82.08	68.99	30.59	424.91	30.6	13.87	70.85	1736
S2	288.74	37.6	176.41	72.95	114.32	173.09	73.37	38.61	239.2	2419
Demand	907	576	445	335	272	431	304	128	757	

5.1. Problem formulation:

Factory S1, Factory S2, Xij = Units moved from source *i* to destination *j*

Using the transport costs shown in the table above, we can write the following equations:

5.2. Cost minimization function:

We add all the values from the table, by multiplying the cost with the quantity:
$90.79X_{11} + 88.21X_{12} + 82.08X_{13} + 68.99X_{14} + 30.59X_{15} + 424.91X_{16} + 30.60X_{17} + 13.87\ X_{18} + 70.85X_{19} + 228.74X_{21} + 37.60X_{22} + 176.41X_{23} + 72.955X_{24} + 114.32X_{25} + 173.09X_{26} + 73.37X_{27} + 38.61X_{28} + 239.2020X_{29}$

5.3. Capacity constraints:

First row: $X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} + X_{17} + X_{18} + X_{19} \leq 1736$
Second row: $X_{21} + X_{22} + X_{23} + X_{24} + X_{25} + X_{26} + X_{27} + X_{28} + X_{29} \leq 2419$

5.4. Demand constraints:

$X_{11} + X_{21} = 907$
$X_{12} + X_{22} = 576$
$X_{13} + X_{23} = 445$
$X_{14} + X_{24} = 335$
$X_{15} + X_{25} = 272$
$X_{16} + X_{26} = 431$
$X_{17} + X_{27} = 304$
$X_{18} + X_{28} = 128$
$X_{19} + X_{29} = 757$
$X_{ij} \geq 0$ for each i and j

5.5. The North West Method

Table 2. North West Method, First selection

From/ To	D 1	D2	D3	D4	D5	D6	D7	D8	D9	Capacity
	90.79	88.21	82.08	68.99	30.59	424.91	30.6	13.87	70.85	
S1	907									829
	288.74	37.6	176.41	72.95	114.32	173.09	73.37	38.61	239.2	
S2										2419
Demand	0	576	445	335	272	431	304	128	757	

First select the angle from the far left of the matrix, where are assigned as many units as possible within the bounds of demand and supply. We allocate 907 units in the first cell that has meet the demand of destination D1 but supply S1 is bigger. The supply capacity of the S1 is more with: 1736-907 = 829 units. This logic explained above is also followed for the other cells.

Table 3. North West Method, Final table of determinations:

From/ To	D 1	D2	D3	D4	D5	D6	D7	D8	D9	Capacity
	90.79	88.21	82.08	68.99	30.59	424.91	30.6	13.87	70.85	
S1	907	576	253							1736
	288.74	37.6	176.41	72.95	114.32	173.09	73.37	38.61	239.2	
S2			192	335	272	431	304	128	757	2419
Demand	907	576	445	335	272	431	304	128	757	

Summary of results of Variables and Values:
X_{11}: 907; X_{12}:576; X_{13}:253; X_{14},X_{15},X_{16},X_{17},X_{18},X_{19}:0
X_{21},X_{22}:0; X_{23}:192; X_{24}:335; X_{25}:272; X_{26}:431; X_{27}:304; X_{28}:128; X_{29}:0

Minimum cost total = 90.97 x 907 + 88.21 x 576 +82.08 x 253 +176.41 x 192 + 72.95 x 335 + 114.32 x 272 + 173.09 x 431 + 73.37 x 304 + 338.61 x 128 + 29.2 x 128 + 239.2 x 757 = ***564811.75***

The number of cells allocated is 10 equivalent to: m + n-1 = 2 + 9-1 = 10, indicating that this solution is not degraded.

Table 4. Least Cost Method, first table

From/ To	D 1	D2	D3	D4	D5	D6	D7	D8	D9	Capacity
	90.79	88.21	82.08	68.99	30.59	424.91	30.6	13.87	70.85	
S1								128		1608
	288.74	37.6	176.41	72.95	114.32	173.09	73.37	38.61	239.2	
S2										2419
Demand	907	576	445	335	272	431	304	0	757	

The minimum transport cost is in cell S1 D8. The maximum that can be determined in this cell is 128 units. Requirement for destination D8 is fulfilled, while factory S1 has a capacity of: 1736 -128 = 1608 units.

Table 5. Least Cost Method, final table of determinations

From/ To	D 1	D2	D3	D4	D5	D6	D7	D8	D9	Capacity
				68.99	30.59				70.85	
	90.79	88.21	82.08			424.91	30.6	13.87		
S1				335	272		304	128	697	1736
						173.09				
	288.74	37.6	176.41	72.95	114.32		73.37	38.61	239.2	
S2	907	576	445			431			60	2419
Demand	907	576	445	335	272	431	304	128	757	

Summary of results of Variables and Values:
X_{11},X_{12},X_{13}:0; X_{14}:335; X_{15}:272; X_{16}:0; X_{17}:304; X_{18}:128; X_{19}:697
X_{21}: 907; X_{22}:576; X_{23}:445; X_{24},X_{25}:0; X_{26}:431; X_{27},X_{28}:0; X_{29}:60

Minimum transport cost = 335 x 68.99 + 272 x 30.59 +304 x 30.6 +128 x 13.87 + 697 x 70.85 + 907 x 288.74 + 576 x 37.6 + 445 x 176.41 + 431 x 173.09 + 60 x 239.2 = 542893.36
The number of cells allocated is 10 equivalent to: m+n-1 = 2+9-1 =10, indicating that this solution is not degraded.

Table 6. Vogel's Method

From/ To	D 1	D2	D3	D4	D5	D6	D7	D8	D9	Capacity	Penal of the row
	90.79	88.21	82.08	68.99	30.59	424.91	30.6	13.87	70.85		
S1										1736	16.72
	288.74	37.6	176.41	72.95	114.32	173.09	73.37	38.61	239.2		
S2										2419	1.01
Demand	907	576	445	335	272	431	304	128	757		
Column Penalty	197.77	50.61	94.33	3.96	83.73	251.82	42.77	24.74	168.35		

The maximum penalty is in the sixth column at 251.82 and the minimum cost Cij is in column C26 = 173.09. The maximum determination we can make in this cell is 431. This satisfies the 6th column and the second row is completely saturated with a difference from 2419 to 1988: (S2 = 2419- 431 = 1988)

Table 7. Vogel's Method, first table

From/ To	D 1	D2	D3	D4	D5	D6	D7	D8	D9	Capacity	Penal of the row
S1	90.79	88.21	82.08	68.99	30.59	424.91	30.6	13.87	70.85	1736	16.72
S2	288.74	37.6	176.41	72.95	114.32	173.09 431	73.37	38.61	239.2	1988	1.01
Demand	907	576	445	335	272	0	304	128	757		
Column Penalty	197.77	50.61	94.33	3.96	83.73	--	42.77	24.74	168.35		

Table 8. Vogel's Method, final table

From/ To	D 1	D2	D3	D4	D5	D6	D7	D8	D9	Capacity
S1	90.79 907	88.21	82.08 72	68.99	30.59	424.91	30.6	13.87	70.85 757	1736
S2	288.74	37.6 576	176.41 373	72.95 335	114.32 272	173.09 431	73.37 304	38.61 128	239.2	2419
Demand	907	576	445	335	272	431	304	128	757	

Summary of results of Variables and Values:
X_{11}: 907; X_{12}:0; X_{13}:72; X_{14}, X_{15}, X_{16}, X_{17}, X_{18}:0; X_{19}:757
X_{21}: 0; X_{22}:576; X_{23}:373; X_{24}:335; X_{25}:272; X_{26}:431; X_{27}:304; X_{28}:128; X_{29}:0

Minimum cost total = 90.97 x 907 + 82.08 x 72 +176.4 x 373 + 72.95 x 335 + 114.32 x 272 + 173.09 x 431 + 73.37 x 304 + 38.61 x 128 + 70.85 x 757 + 37.6 x 576 = 386893.17
The number of cells allocated is 10 equivalent to: m+n-1 = 2+9-1 =10, indicating that this solution is not degraded.

Table 9. Summary table of the three methods: NWC, LCM, Vogel's

Variables	Value of NWC	Value of LCM	VOGEL'S Value
X_{11}	907	0	907
X_{12}	576	0	0
X_{13}	253	0	72
X_{14}	0	335	0
X_{15}	0	272	0
X_{16}	0	0	0
X_{17}	0	304	0
X_{18}	0	128	0
X_{19}	0	697	757
X_{21}	0	907	0
X_{22}	0	576	576
X_{23}	192	445	373
X_{24}	335	0	335
X_{25}	272	0	272
X_{26}	431	431	431
X_{27}	304	0	304
X_{28}	128	0	128
X_{29}	0	60	0
Minimum cost for each method	564811.75	542893.36	*386893.17*

***All company costs are 10 ^ 3**

Having solved the problem with all three methods then it turns out that the most efficient method is the Vogel's method. This is the method which is chosen as the most optimal as it results in a lower cost than the other two methods.

Table 10. Case Study transport table

From Source / To Destination	Dest 1	Dest 2	Dest 3	Dest 4	Dest 5	Dest 6	Dest 7	Dest 8	Dest 9	Capacity
Source 1	90.79 907	88.21	82.08 72	68.99	30.59	424.91	30.6	13.87	70.85 757	1736
Source 2	288.74	37.6 576	176.41 373	72.95 335	114.32 272	173.09 431	73.37 304	38.61 128	239.2	2419
Demand	907	576	445	335	272	431	304	128	757	

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6. Conclusions and future work

The paper considered a real problem of linear programming in detail by taking an example in an Albania company. For the company under consideration the problem of minimizing transportation costs was solved by solving 3 methods: The North West Corner Method, the Least Cost Method, and the Vogel's Approximation Method. The calculations showed that based on the demand from the 9 geographical sites (detinations) and the capacity offered by the two manufacturing plants (sources), the most optimal solution turns out to be the one obtained by the Vogel's method at a minimum cost of 386893.17. Finally, the demand for the 3 geographical regions is covered by the S1 production plant, while the demand for the other 6 regions is covered by the S2 production plant.

In the future we can continue to apply the other steps of transportation methods in order to update and optimise the solution. Also, we can continue the research with experimenting different methods of operational research like simplex method, etc., and try to maximise the profit in an industrial enterprise.

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